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Performance of LM-type unit root tests with trend break: A bootstrap approach

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Abstract

Using the bootstrap approach, we study the finite-sample properties of the Lagrange Multiplier (LM) unit root tests when level shifts are allowed under the null hypothesis. Bootstrapped critical values support the invariance property of the LM tests. Applying two LM-type tests to the Nelson–Plosser data, we find less evidence against the unit root null than that given by Zivot and Andrews [Zivot, E. and Andrews, D.W.K. (1992), "Further Evidence of the Great Crash, the Oil Price Shock and the Unit Root Hypothesis," Journal of Business and Economic Statistics 10, 251–270.] when level shifts are allowed under the null.

Keywords: Lagrange Multiplier unit root test; Bootstrap; Structural change

JEL classification: C12; C22

1. Introduction

Since the seminal work of Nelson and Plosser (1982), testing for unit roots in macroeconomic time series has attracted great interest. Applying the Dickey and Fuller (DF) unit root tests to 14 U.S. macroeconomic time series, Nelson and Plosser found that the unit root null hypothesis was not rejected in 13 cases. Consequently, Perron (1989) challenged the said findings by showing that the unit root tests lack power when there is a structural break in the trend. Perron's paper stimulated considerable interest in studying the effect of structural breaks in unit root tests. In relation to this, one line of research is on unit

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root tests with the endogenous selection of a break date. Examples of this include studies by Banerjee et al. (1992), Zivot and Andrews (1992), Lumsdaine and Papell (1997), and Perron (1997), which are all DF-type tests. Meanwhile, recent studies indicate the potential problems associated with these DF-type tests. For example, Nunes et al. (1997) and Lee and Strazicich (2001) found severe size distortions associated with the Zivot–Andrews-type of unit root tests when a structural break is present but is ignored in the null hypothesis. Furthermore, Lee and Strazicich (2001) found that the asymptotic null distributions of the DF-type endogenous break test statistics are affected by nuisance parameters indicating the magnitude and location of the break, and that Zivot–Andrews-type tests give spurious rejections in the presence of structural breaks.

Another line of research is based on the Lagrange Multiplier (LM) principle which was initially proposed by Schmidt and Phillips (1992; SP hereafter). Several studies have also shown that the SP-type tests with a structural break possess desirable properties that are not found in the DF-type tests. For example, Amlser and Lee (1995) found that the asymptotic null distributions of the LM test statistics are unaffected by the incorrect placement of structural breaks. Related results were also proven by Nunes (2003), and Lee and Strazicich (2003, 2004). However, the asymptotic distribution may not provide a reliable guide to finite-sample behavior. One alternative to the approximate finite-sample distribution of a test statistic is the bootstrap method.[1] Bootstrapped critical values are specific to the sample size, break pattern, and the lag order of the time series. Thus, utilizing bootstrapped critical values can help obtain more reliable statistical inferences. In accordance with this, the aim of this paper is to compare the finite-sample properties of two recently developed LM-type unit root tests with a break using the Nelson–Plosser data.

Our paper is organized as follows. Section 2 presents the models and test statistics. Section 3 discusses the bootstrap method and presents the bootstrapped critical values, empirical sizes, and power simulations. Finally, Section 4 presents the conclusions.

2. The model and LM-type unit root tests

2.1. Specification of the model

As used by Nunes (2003),[2] we consider the following data generating process (DGP) for a time series y_t:

$$y_t = \delta_1 + Z_t\delta + x_t, \tag{1}$$

$$x_t - \alpha x_{t-1} = \varepsilon_t, \tag{2}$$

where Z_t is a vector of exogenous variables, and $\varepsilon_t \sim \text{iid}N(0,\sigma^2)$. The unit root corresponds to $\alpha = 1$. In particular, three models are considered. Model 1 allows for a break in the intercept with

$$Z_t = (t, DU_t), \tag{3}$$

[1] Diebold and Chen (1996) and Rayner (1990), for example, suggest that the bootstrap may be able to provide useful information in many hypothesis-testing situations including the one encountered here, i.e., the distribution of the test statistics is known in large samples but is unknown in small samples.

[2] Lee and Strazicich (2004) considered only Model 1.

where $DU_t = 1$ $(t > \mathrm{TB})$, and $1(\cdot)$ is the indicator function. TB denotes an unknown date when a break occurs. In Model 2, a break is present in both intercept and slope; hence,

$$Z_t = (DU_t, t, DT_t), \tag{4}$$

where $DT_t = 1$ $(t > TB)(t - TB)$. Model 3 allows for a break in slope and

$$Z_t = (t, DT_t). \tag{5}$$

The LM unit root test statistic is estimated by regression according to the LM principle as follows. Under the null hypothesis $\alpha = 1$, the restricted maximum likelihood estimators of δ in (1), denoted by $\tilde{\delta}$, are obtained by estimating the following equation by OLS:

$$\Delta y_t = \Delta Z_t \delta + e_t, \tag{6}$$

where ΔZ_t is the first difference of the regressors Z_t. We write the detrended series

$$\widetilde{S}_t \text{ as } \widetilde{S}_t = y_t - \tilde{\delta}_1^* - Z_t \tilde{\delta}, \tag{7}$$

where $\tilde{\delta}_1^* = y_1 - Z_1 \tilde{\delta}$, with y_1 and Z_1 denoting the first observations of y_t and Z_t, respectively.[3]

The LM-type unit root test statistic can be estimated from the test regression as

$$\Delta \widetilde{S}_t = \Delta Z_t \delta + \phi \widetilde{S}_{t-1} + e_t. \tag{8}$$

To allow the existence of autocorrelation in the errors, we can estimate an augmented regression specified as

$$\Delta \widetilde{S}_t = \Delta Z_t \delta + \phi \widetilde{S}_{t-1} + \sum_{1}^{k} c_j \Delta \widetilde{S}_{t-j} + e_t, \tag{9}$$

where k is the augmented lag order. The LM-type test statistic is given by the t-statistic for testing the null of $\phi = 0$, and is defined as $t_\phi(m, TB)$, where m denotes Models 1, 2, and 3 as described in (3)–(5), respectively, while TB is the break date.

2.2. Break selection methods and estimation of LM test statistics

To implement the LM-type unit root tests when a break is allowed under the null hypothesis, we require the estimation of the break date (TB). Two alternative break date selection methods are considered. In the first case, we follow that of Lee and Strazicich (2004) by choosing the break date over a range of possible break points, where the t-statistic testing the null of a unit root is minimized in Eq. (9).[4] The break date selected in this way is denoted as $\mathrm{TB}(t_\phi)$, and its corresponding t-statistic is the minimum LM test statistic which is to be denoted as $t_\phi(m, \mathrm{TB}(t_\phi))$. As an alternative, we also consider the break date selection methods proposed by Nunes (2003) in which the break date is chosen by maximizing the absolute value of

[3] See Schmidt and Phillips (1992) for details.

[4] The number of k augmented terms is determined jointly with the selected break date. The maximum value of k is set at 8.

the t-statistics for ΔDU_t in regression (9)[5] for Model 1, and by maximizing the F-statistic for ΔDU_t and ΔDT_t in (9) for Model 2. The estimated break date and the t-statistics are denoted as $\hat{TB}$ and $t_{\phi}(m,\hat{TB})$, respectively.

When these tests are applied to the Nelson–Plosser data,[6] the results (Table 1) show that in the two LM-type tests, 11 out of the 13 series have different estimated break dates. The salient feature of these results is the larger t-statistics (in absolute value) estimated by the minimum LM test as compared to those obtained by the Nunes LM test. Using the minimum LM test, we can reject the unit root null for money stock, stock prices, and real wages at the 5% level, and reject that for industrial production and consumer prices at the 10% level. However, we can only reject the unit root null for the money stock series using the Nunes LM test at the 5% level, and reject that for employment and real wages at the 10% level. As compared to the findings of Nunes et al. (1997), our results provide a weaker evidence against the unit root hypothesis for the real-GNP series, but there is a stronger evidence obtained by our study against the unit root hypothesis for the money-stock, real-wage, and stock-price series.

3. Bootstrapping the critical values

3.1. Bootstrapped critical values with a break

The tests on the unit root null hypothesis with a break are based on the bootstrapped critical values which are obtained by simulating the fitted models described in Section 2.2 using the bootstrap method with 5000 replications.[7] The bootstrapped critical values for the two alternative LM-type tests with a break are shown in Table 2 under the column DGP: 1 break. When no breaks are allowed under the null, the bootstrapped critical values are the same for the two LM-type tests as shown under the column DGP: 0 breaks of Table 2.

The critical values displayed in Table 2 for the 11 Model-1 series indicate that the critical values with a break are nearly identical to their corresponding no-break counterparts in the two LM-type tests. They confirm the asymptotic invariance property of the endogenous-break LM test statistics as discussed by Lee and Strazicich (2003). However, the critical values of the two Model-2 series presented in Table 2 give different implications. The Model 2 critical values indicate that the Nunes LM test is more sensitive to the presence of breaks than the minimum LM test. For example, at the 5% level, the Model 2 critical value (with a break under the null) of the Nunes LM test is 7.6% larger (in absolute value) than its corresponding no-break critical value for stock prices, while that of the minimum LM test is 2.2% larger (in absolute value) than its no-break counterpart. For the real-wage series, at the 5% level, the Model 2 critical value (with a break under the null) of the Nunes LM test is only slightly (<1%) larger (in absolute value) than the corresponding no-break critical value, while the critical values of the minimum LM test

[5] Similar to the minimum LM test, the number k is determined jointly with the selected break date.

[6] The choice of break models follows that of Perron, and Zivot and Andrews.

[7] The bootstrap procedure consists of five steps: (i) Estimate Eq. (6) with the original data, then select the break date by means of the alternative break selection criteria. (ii) Using the estimated coefficients in step 1, we compute the restricted residuals denoted as $\hat{e}_t^{\mathrm{r}}$, based on (7) under the null $\alpha=1$. (iii) Draw random samples from $\hat{e}_t^{\mathrm{r}}$ with replacement, and each size must be equal to the actual size of the series. Then construct pseudo samples y_t^* based on $y_t^*=\delta_1+Z_t\delta+x_t, x_t-\alpha x_{t-1}=e_t^{*\mathrm{r}}$. (iv) Run the following regression: $\Delta\tilde{S}_t^*=\Delta Z_t\delta+\phi\tilde{S}_{t-1}^*+\sum_{j=1}^{k}c_j\Delta\tilde{S}_{t-j}^*+e_t$ by utilizing the pseudo samples y_t^*, and compute the t-statistic for testing $\phi=0$. (v) Repeat steps (iii) and (iv) 5000 times, and obtain the critical values with a break under the null from the sorted vector of 5000 t-statistics.

Table 1
Empirical results of alternative LM unit root tests using the Nelson–Plosser time series

Series	Model type(m)	LM test statistics					
		$t_\phi(m,\mathrm{TB}(t_\phi))$ [a]	Break year	k	$t_\phi(m,\hat{\mathrm{TB}})$ [b]	Break year	k
Real GNP	1	−3.26	1920	1	−2.60	1931	1
Nominal GNP	1	−2.96	1921	1	−2.67	1920	5
Real per capita GNP	1	−3.19	1920	1	−2.43	1931	1
Industrial production	1	−3.66*	1937	3	−2.86	1931	5
Employment	1	−3.28	1931	7	−3.28*	1931	7
GNP deflator	1	−2.63	1921	1	−2.63	1920	5
Consumer prices	1	−3.79*	1916	4	−2.65	1920	3
Wages	1	−3.46	1920	7	−2.19	1917	6
Money stock	1	−3.97**	1931	7	−3.97**	1931	7
Velocity	1	−2.19	1893	1	−1.54	1945	0
Interest rate	1	−1.36	1953	3	−1.20	1967	3
Stock prices	2	−5.24**	1951	1	−3.16	1931	7
Real wages	2	−5.10**	1939	3	−4.89*	1940	3

For Model 1 ($m=1$), it is the maximal absolute value of the t-statistic of the break dummy ΔDU_t. For Model 2 ($m=2$), it is the maximal absolute value of the F-statistic on break dummies ΔDU_t and ΔDT_t.
k is the estimated lag order in (9).
** and * denote significance at the 5% and 10% level, respectively.
[a] $t_\phi(m,\mathrm{TB}(t_\phi))$ = Minimum LM test of Lee and Strazicich (2004).
[b] $t_\phi(m,\hat{\mathrm{TB}})$ = LM test statistics of Nunes (2003).

(with a break under the null) are nearly identical to their corresponding no-break critical values. These results confirm the findings of Lee and Strazicich (2003) that the minimum LM test is not invariant for Model 2, but it is nearly so.

3.2. Finite-sample size and power analysis

We now examine the finite-sample size and power of the LM-type tests using the bootstrapped critical values with a break. Table 2 shows that for the 11 Model-1 series, no size distortion is found for the two LM-type tests, as reflected by the close to 5% size values. However, for the two Model-2 series, both tests become undersized when a break in both the intercept and slope is allowed.

Next, we analyze the power of the LM-type tests. The last column of Table 2 presents powers against the $\alpha=0.8$ alternative. The minimum LM statistics have better power than the Nunes statistics, particularly when there is no break under the null. However, in the presence of a break, the deterioration in power is much larger for the minimum LM test than that of the Nunes test. Power deterioration is found in 11 out of the 13 series using the minimum LM test, and it is particularly severe for real GNP and real per capita GNP.

4. Conclusion

Recent studies show that the LM-type tests with a structural break possess desirable properties not found in the DF-type tests. Using the bootstrap method, we studied the finite-sample properties of two alternative LM-type unit root tests allowing a break under the null hypothesis. Our major findings show that first, the bootstrapped critical values are not affected by a break for Model 1 (with a break in the

Table 2
Bootstrapped critical values, empirical size and power

Series	Model	Break selection scheme	DGP: 0 breaks Regression (9)			DGP: 1 break			Size [a] (%)	Power [b] (%) DGP:	
			1%	5%	10%	1%	5%	10%		0 breaks	1 break
Real GNP	1	Min LM [c]	−4.70	−4.07	−3.72	−4.67	−4.07	−3.73	5.5	25.1	18.1
		Max |tb| [c]	−4.25	−3.46	−3.10	−4.14	−3.45	−3.08	4.7	16.9	16.0
Nominal GNP	1	Min LM	−4.75	−4.04	−3.70	−4.75	−4.05	−3.70	5.3	27.1	26.6
		Max |tb|	−4.15	−3.41	−3.07	−4.13	−3.43	−3.08	4.6	17.1	14.7
Real per capita GNP	1	Min LM	−4.81	−4.08	−3.74	−4.77	−4.07	−3.73	5.6	23.2	15.4
		Max |tb|	−4.23	−3.49	−3.16	−4.10	−3.42	−3.11	5.1	14.6	16.1
Industrial production	1	Min LM	−4.50	−3.84	−3.50	−4.55	−3.87	−3.52	4.4	66.1	52.6
		Max |tb|	−3.86	−3.30	−2.97	−3.91	−3.26	−2.92	5.3	39.0	41.1
Employment	1	Min LM	−4.62	−3.98	−3.61	−4.63	−4.00	−3.65	4.9	39.5	33.4
		Max |tb|	−4.07	−3.40	−3.07	−4.08	−3.40	−3.05	5.1	25.2	23.4
GNP deflator	1	Min LM	−4.68	−3.99	−3.66	−4.72	−3.99	−3.64	5.0	7.0	7.6
		Max |tb|	−4.09	−3.37	−3.02	−4.04	−3.34	−3.00	4.4	6.1	6.2
Consumer prices	1	Min LM	−4.49	−3.86	−3.51	−4.57	−3.89	−3.55	4.3	7.1	9.3
		Max |tb|	−3.87	−3.25	−2.92	−3.86	−3.25	−2.94	5.2	5.4	3.3
Wages	1	Min LM	−4.70	−4.06	−3.69	−4.74	−4.05	−3.71	5.2	30.0	25.0
		Max |tb|	−4.16	−3.45	−3.10	−4.05	−3.35	−3.03	6.1	19.0	22.7
Money stock	1	Min LM	−4.66	−3.98	−3.64	−4.68	−3.93	−3.63	4.9	38.0	29.6
		Max |tb|	−4.04	−3.36	−3.00	−3.98	−3.34	−3.00	4.9	22.9	26.8
Velocity	1	Min LM	−4.58	−3.91	−3.53	−4.62	−3.91	−3.55	4.7	41.9	34.9
		Max |tb|	−4.03	−3.33	−3.02	−4.08	−3.40	−3.04	4.8	30.2	25.1
Interest rate	1	Min LM	−4.73	−3.98	−3.68	−4.69	−4.00	−3.69	4.8	32.8	31.9
		Max |tb|	−3.99	−3.38	−3.04	−4.05	−3.33	−3.01	4.9	21.4	21.7
Stock prices	2	Min LM	−5.48	−4.93	−4.59	−5.40	−4.82	−4.53	4.7	28.0	24.8
		Max |tb|	−5.25	−4.48	−4.08	−4.91	−4.14	−3.79	4.7	20.3	22.7
Real wages	2	Min LM	−5.71	−5.08	−4.74	−5.68	−5.06	−4.76	4.3	14.4	12.4
		Max |tb|	−5.53	−4.93	−4.59	−5.47	−4.90	−4.51	4.3	13.1	12.3

[a] The size is computed by using the 5% critical values from the DGP with one break.
[b] The power is computed by assuming $\alpha = 0.8$ under the alternative using the 5% critical values.
[c] See notes *a* and *b* to Table 1.

intercept), thus confirming the invariance property of the endogenous-break LM tests shown by Lee and Strazicich (2003). The minimum LM test is not invariant for Model 2 (with breaks in both the intercept and slope), but it is nearly so. Second, for the 11 Model-1 series, both the minimum LM and the Nunes LM test have sizes close to 5%. However, for the two Model-2 series, stock prices and real wages, both LM-type tests are undersized. The minimum LM test, on the other hand, is more powerful than the Nunes LM test in all the cases we examined. Finally, we found that when allowing for a break in the intercept (Model 1), the two LM-type unit root tests provide less evidence[8] against the unit root null hypothesis

[8] For Model 1, the minimum LM test rejects the unit-root null at the 5% level for one series and at the 10% level for two series, and the Nunes LM test rejects the unit-root null at the 5% level for one series and at the 10% level for one series. However, the ZA test rejects the unit-root null at the 1% level for one series, at the 5% level for two series, and at the 10% level for one series. For Model 2, the test results are mixed. The Nunes LM test rejects the unit-root null at the 10% for one series which is in accordance with the ZA test, while the minimum LM test rejects the unit-root null at the 5% for two series.

than that given by the Zivot–Andrews (ZA)-type endogenous break unit root tests, possibly indicating the problem of over-rejections in the DF-type tests.

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